Homework 281 __ **BUNKER HILL COMMUNITY COLLEGE** **MAT - 281**

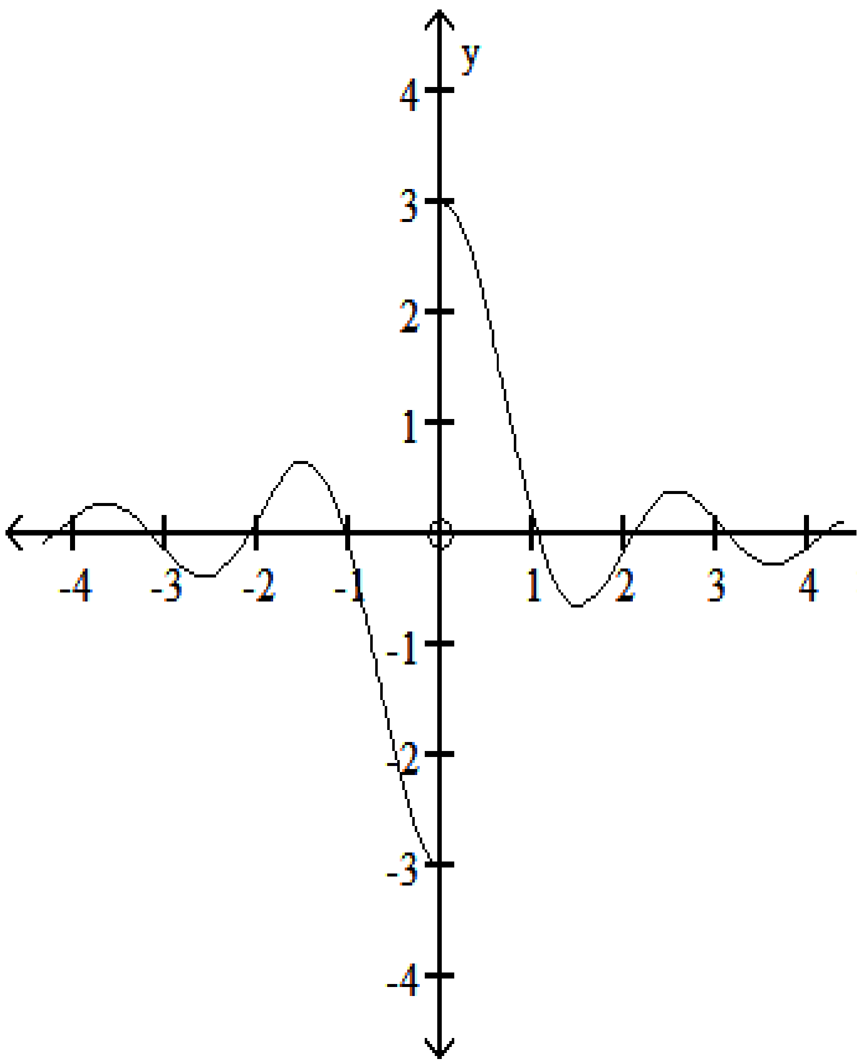
Chapter 1 and 3.5

Name________________________________

PLEASE SHOW ALL YOUR WORK AS NEATLY A S POSSIBLE :
SHOW ALL YOUR WORK TO RECEIVE FULL POINTS :

Use the graph to evaluate the limit. Does the limit exist ? Why?

1) $\lim_{x \to 0^-} f(x) =$ $\lim_{x \to 0^+} f(x) =$



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$\lim x \to 0^- f(x) = -3$
$\lim x \to 0^+ f(x) = 3$

The limit does not exist because there is a jump discontinuity at 0 i.e. left-hand limit ≠ right-hand limit.

2. Find the limits:

a) $\lim_{x\to 0} \frac{x-\sin(x)}{x}$

$$\lim_{x\to 0}\left(\frac{x-\sin(x)}{x}\right) = \lim_{x\to 0}\left(1-\frac{\sin(x))}{x}\right)$$

= by squeeze theorem, $\lim_{x\to 0}\frac{\sin(x)}{x} = 1$

= 1-1 = 0

$\lim_{x\to 0}\frac{x-\sin(x)}{x} = 0$

b) $\lim_{x\to 0}\frac{\sin(2x)}{2x^2+x}$

$$\lim_{x\to 0}\left(\frac{\sin(2x)}{2x^2+x}\right) = \lim_{x\to 0}\left(\frac{2x\,\frac{\sin(2x)}{2x}}{2x^2+x}\right)$$

$=\lim_{x\to 0}\left(\frac{2\,\frac{\sin(2x)}{2x}}{2x+1}\right) = 2 * \lim_{x\to 0}\left(\frac{\frac{\sin(2x)}{2x}}{2x+1}\right)$

by squeeze theorem, $\lim_{x\to 0}\frac{\sin(x)}{x}$, thus $\lim_{x\to 0}\left(\frac{\sin(2x)}{2x}\right) = 1$

$= \frac{2*1}{1} = 2$

$\lim_{x\to 0}\frac{\sin(2x)}{2x^2+x} = 2$

3. Find the limits:

a) $\lim\limits_{x \to -\pi} \sqrt{x+9}\cos(x+\pi)$

$= \sqrt{-\pi+9}\cos(-\pi+\pi)$

$= \sqrt{-\pi+9} \approx 2.42041$

b) $\lim\limits_{x \to \infty} \frac{x-\cos(x)}{x}$

$\lim\limits_{x \to \infty} \frac{x-\cos(x)}{x} = \lim\limits_{x \to \infty} (1 - \frac{\cos(x)}{x})$

$= \lim\limits_{x \to \infty} (1) - \lim\limits_{x \to \infty} (\frac{\cos(x)}{x})$

But the limit of a constant is equal to the constant.

$\frac{-1}{x} \leq (\frac{\cos(x)}{x}) \leq \frac{1}{x}$

$= \lim\limits_{x \to \infty} (\frac{-1}{x}) \leq \lim\limits_{x \to \infty} (\frac{\cos(x)}{x}) \leq \lim\limits_{x \to \infty} (\frac{1}{x})$

$0 \leq (\frac{\cos(x)}{x}) \leq 0$

Thus $\lim\limits_{x \to \infty} (\frac{\cos(x)}{x}) = 0$

$= 1 + 0$

$\lim\limits_{x \to \infty} \frac{x-\cos(x)}{x} = 1$

4. Find the limits:

a) $\lim\limits_{x \to 0} \frac{1-\cos(x)}{x^2}$

$$\lim_{x \to 0} \left(\frac{1-\cos(x)}{x^2}\right) = \lim_{x \to 0} \left(\frac{(1-\cos(x))}{x^2} * \frac{1+\cos(x)}{1+\cos(x)}\right)$$

$= \lim\limits_{x \to 0} \left(\frac{1-\cos^2(x)}{x^2(1+\cos(x))}\right)$

$$= \lim_{x\to 0}\left(\frac{\sin^2(x)}{x^2(1+\cos(x))}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sin^2(x)}{x^2}\right) * \lim_{x\to 0}\left(\frac{1}{(1+\cos(x))}\right)$$

Taking $\lim_{x\to 0}\left(\frac{\sin^2(x)}{x^2}\right) = \lim_{x\to 0}\left(\frac{\sin(x)}{x}\right)*\lim_{x\to 0}\left(\frac{\sin(x)}{x}\right) = 1*1 = 1$

$$= 1* \frac{1}{(1+\cos(0))} = \frac{1}{2}$$

$$\lim_{x\to 0}\frac{1-\cos(x)}{x^2} = \frac{1}{2}$$

b) $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$

$$= \lim_{x\to 4}\left(\frac{\sqrt{x}-2}{x-4}\right) = \lim_{x\to 4}\left(\frac{\sqrt{x}-2}{\sqrt{x^2}-2^2}\right)$$

$$= \lim_{x\to 4}\left(\frac{\sqrt{x}-2}{(\sqrt{x}-2)(\sqrt{x}+2)}\right)$$

$$= \lim_{x\to 4}\left(\frac{1}{\sqrt{x}+2}\right)$$

$$= \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \frac{1}{4}$$

5. Find the following limits:

a) $\lim_{x\to \pi}\frac{\sin(x)}{2+\cos(x)}$

Substituting the variable with the value:

$$= \frac{\sin(\pi)}{2+\cos(\pi)}$$

$$= \frac{0}{1}$$

$$\lim_{x\to \pi}\frac{\sin(x)}{2+\cos(x)} = 0$$

b) $\lim_{x\to 1}\frac{\sqrt{2x+1}-\sqrt{3}}{x}$

Substituting x with 1:

$$= \frac{\sqrt{2+1} - \sqrt{3}}{1} = 0$$

$$\lim_{x \to 1} \frac{\sqrt{2x+1} - \sqrt{3}}{x} = 0$$

6. Find the limit:

$$\lim_{x \to \infty} \left(4x - \sqrt{16x^2 - x}\right)$$

$$\lim_{x \to \infty} \left(4x - \sqrt{16x^2 - x}\right) = \lim_{x \to \infty} \frac{x}{4x - \sqrt{16x^2 - x}}$$

$$= \lim_{x \to \infty} \frac{x}{x \frac{4x - \sqrt{16x^2 - x}}{x}}$$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{16 - \frac{1}{x}} + 4}$$

Substituting x with ∞:

$$\frac{1}{\infty} = 0$$

Thus $\lim_{x \to \infty} \frac{1}{\sqrt{16 - \frac{1}{x}} + 4} = \frac{1}{\sqrt{16} + 4} = \frac{1}{8}$

$$\lim_{x \to \infty} \left(4x - \sqrt{16x^2 - x}\right) = \frac{1}{8}$$

7. Evaluate the limit: $\lim_{x \to -4} \frac{\frac{1}{x} + \frac{1}{4}}{x+4}$

Solving for the numerator: $\frac{1}{x} + \frac{1}{4} = \frac{4+x}{4x}$

$$\lim_{x \to -4} \frac{\frac{4+x}{4x}}{x+4} = \lim_{x \to -4} \frac{x+4}{4x(x+4)}$$

$$= \frac{1}{(4*4)} = \frac{-1}{16}$$

$$\lim_{x \to -4} \frac{\frac{1}{x} + \frac{1}{4}}{x+4} = \frac{-1}{16}$$

8. Find the limit: $\lim_{x \to 7}\left(\frac{\sqrt{x+2}-3}{x-7}\right)$

Rationalizing the numerator:

$$\frac{\sqrt{x+2}-3}{x-7} * \frac{\sqrt{x+2}+3}{\sqrt{x+2}+3} = \frac{x-7}{x-7(\sqrt{x+2}+3)}$$

Thus $\lim_{x \to 7}\left(\frac{\sqrt{x+2}-3}{x-7}\right) = \lim_{x \to 7}\left(\frac{1}{\sqrt{x+2}+3}\right) = \frac{1}{\sqrt{7+2}+3} = \frac{1}{6}$

$$\lim_{x \to 7}\left(\frac{\sqrt{x+2}-3}{x-7}\right) = \frac{1}{6}$$

9. Find the limit: $\lim_{x \to 1}\left(\frac{x^3 - 1}{x^2 - 1}\right)$

$$\lim_{x \to 1}\left(\frac{x^3 - 1}{x^2 - 1}\right) = \lim_{x \to 1}\left(x + \frac{(x - 1)}{(x^2 - 1)}\right)$$

$$= \lim_{x \to 1}\left(x + \frac{(x - 1)}{(x - 1)(x+1)}\right) = \lim_{x \to 1}\left(x + \frac{1}{(x+1)}\right)$$

Substituting x with 1:

$$1 + \frac{1}{(1+1)} = \frac{3}{2}$$

$$\lim_{x \to 1}\left(\frac{x^3 - 1}{x^2 - 1}\right) = \frac{3}{2}$$

10. Given that f(x) = $\frac{1}{x-1}$

Find the instantaneous rate of change.

$$\lim_{h \to 0}\left(\frac{\frac{1}{(x+h)-1} - \frac{1}{x-1}}{h}\right)$$

Solving the numerator:

$$\frac{1}{(x+h)-1} - \frac{1}{x-1} = \frac{-h}{x-1(h+x-1)}$$

$$\lim_{h\to 0}\left(\frac{\frac{1}{(x+h)-1}-\frac{1}{x-1}}{h}\right) = \lim_{h\to 0}\left(\frac{-1}{(x+1)(h+x-1)}\right) = \frac{-1}{(x+1)(x-1)} = \frac{-1}{(x-1)^2}$$

The instantaneous rate of change is: f'(x) = $\frac{-1}{(x-1)^2}$